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PIEZOELECTRIC CRYSTAL OSCILLATORS APPLIED TO THE PRECISION MEASUREMENT OF THE VELOCITY OF SOUND IN AIR AND CO_2 AT HIGH FREQUENCIES.

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1. Reference to Previous Paper.— In a previous paper [1] I have described a method of producing sustained high-frequency electric and mechanical vibrations by a novel combination of a plate of piezoelectric crystal with a thermionic vacuum tube, and have shown how to employ the apparatus in the calibration of wavemeters. These vibrations are of extraordinary constancy as to frequency so that it has seemed desirable to apply the apparatus to other measurements.

The present account describes the precision measurement of the velocity of sound at high frequency.

2. Preparation of the Piezoelectric Crystal Plate.— Quartz was used as the piezoelectric substance. A plate was cut from the

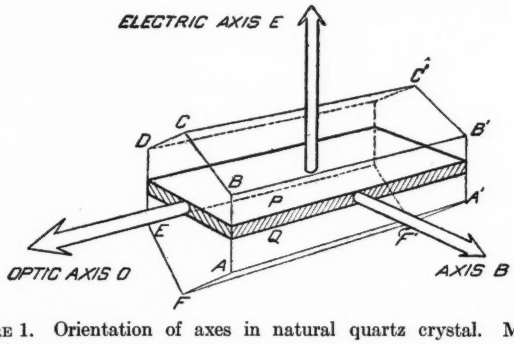

Figure 1. Orientation of axes in natural quartz crystal. Method of sectioning.

natural crystal of quartz with the orientation suggested by P. and J. Curie,[2] as is here shown in Figure 1. After the natural crystal had

[1] G. W. Pierce, Piezoelectric Crystal Resonators and Crystal Oscillators Applied to the Precision Calibration of Wavemeters, Proc. Am. Acad. of Arts and Sciences, **59**, No. 4 (1923). (Reprints may be purchased from the Librarian of the Academy, 28 Newbury Street, Boston, Massachusetts.)

[2] Pierre and Jacques Curie, Comptes Rendus, **91**, 383 (1880); also, Oeuvres de Pierre Curie, Paris, 1908.

been trued up by crosswise cuts so as to form a prism with the hexagonal ends ABCDEF and A′B′C′. . F′, which are respectively perpendicular to the natural edges AA′, BB′ of the crystal, a rectangular slab was obtained from the prism by two parallel cuts P and Q, which are perpendicular to a natural face such as ABB′A′.

3. Mounting of Plate Between Electrodes.— The rectangular slab so obtained has three axes represented in the diagrams by arrows: the *optic axis O* (parallel to the lengthwise natural edges of the crystal), the *electric axis E* (parallel to two opposite natural faces of the crystal), and the *third axis B* (perpendicular to the optic axis and the electric axis). This slab is placed between two metal electrodes M' and M'', as shown in Figure 2. The crystal may rest on M'', while M' may best

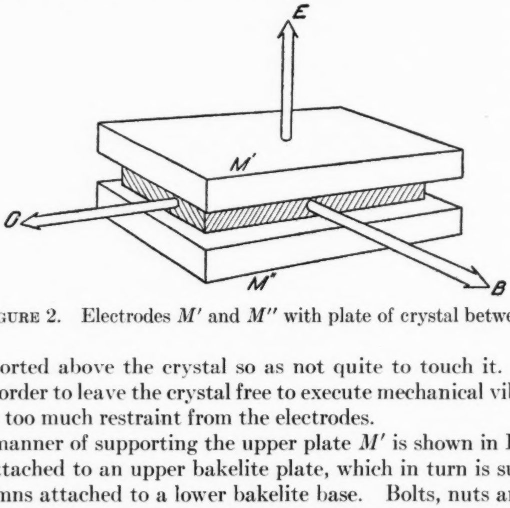

FIGURE 2. Electrodes M' and M'' with plate of crystal between.

be supported above the crystal so as not quite to touch it. This is done in order to leave the crystal free to execute mechanical vibrations, without too much restraint from the electrodes.

The manner of supporting the upper plate M' is shown in Figure 3. M' is attached to an upper bakelite plate, which in turn is supported on columns attached to a lower bakelite base. Bolts, nuts and spiral springs, as shown, permit the adjustment of the clearance between M' and the crystal slab. In Figure 3, the axis B points toward or away from the observer.

Another method of mounting the crystal slab (which is here represented as circular) is shown in Figure 4, in which the plates M' and M'' are in vertical planes, and the plate M'' is perforated with a hole about 1 cm. in diameter, so as to permit the radiation of sound through this

hole. In this figure the plate M'' is in the form of a spring clamp and may rest upon the crystal with sufficient pressure to hold the crystal in place, or if desired, the pressure may be removed by the screw S and the crystal supported independently by a small shelf below.

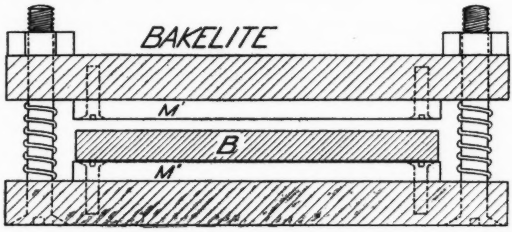

FIGURE 3. Mounting of plate of crystal in clamp with adjustable electrode distances. This arrangement for emitting and receiving sound of frequency determined by dimension in direction of axis B.

In Figure 4, the electric axis E points through the perforate electrode M'', while the axes B and O are in the plane of the crystal slab with orientation that is immaterial for the present purposes.

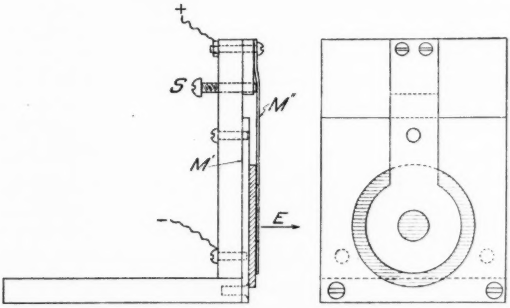

FIGURE 4. Side view and front view of crystal plate mounted in a vertical plane having a perforate electrode for emitting and receiving sound of frequency determined by thickness of plate in direction of electric axis E.

4. The Piezoelectric Action.— As to the action of the piezoelectric crystal, attention is called to the discovery of Curie that a difference of electrical potential of proper sign established between electrodes such as the plate M' and M'' (that is, an electric force of

proper sign established in the direction of the electric axis E of Fig. 2) causes the piezoelectric quartz crystal to expand along the axis E and contract along the axis B. No change occurs along the optic axis O. A reversal of the electric force causes a reversal of these expansions and contractions. Curie also found that an expansion along E or contraction along B produced by an external mechanical pressure developed an electromotive force between the plates (i.e., along E) opposite in sign to that which would produce the given expansion. That is to say, an electric field along E produces distortion of the crystal in directions E and B, and a distortion so produced in these directions reacts to diminish the field along E. In case of alternating forces the amplitude and phase of these effects depend upon the mechanical constants of the crystal and upon the frequency of the applied e.m.f.

Langevin [3] with great ingenuity showed how to use such a piezoelectric crystal body as a source of sound particularly in water, and Cady [4] made a thorough and beautiful investigation of the crystal oscillators and crystal resonators, and adapted them to use as constants of electrical frequency.

5. The Electric Circuits for Producing Sustained Vibrations. — In my work above cited I showed a simple form of connections of the crystal vibrator to a thermionic vacuum tube so as to produce sustained electrical and mechanical vibrations of the system with a period determined by the dimensions of the slab of crystal and independent of the electrical constants of the circuit. Cady had already described a means of doing this by a different type of circuit, which is described in his paper above referred to in the Radio Institute Proceedings. My circuit which for some purposes has certain advantages is illustrated in Figure 5. The piezoelectric crystal vibrator marked "crystal" has one of its electrodes connected to the plate P and the other electrode connected to the grid G of a thermionic vacuum tube, having a filament F heated by the battery marked "*A Bat.*" The plate is supplied with current by the battery marked "*B Bat.*" A microammeter, A, and a telephone "*Tel*" shunted by a bypass condenser C are included in the plate circuit. The element marked "*load*" in the figure was described in my original paper as a resistance of about 30,000 ohms or a large inductance, say 20 milhenries. This

[3] Langevin, Brit. Pat. Specifications, N.S., **457**, No. 145,691 (1920).
[4] W. G. Cady, The Piezoelectric Resonator, Proc. Inst. Radio Engineers, **10**, 83 (1922).

system produces sustained electric oscillations in the circuit and mechanical oscillations of the crystal vibrator with a period of one mode of natural mechanical vibration of the crystal body. This period in my original investigation was the mechanical period of compression and recovery of the crystal in its shortest dimension (which was along the electric axis). Such a system radiates sound in the direction of the electric axis *E*.

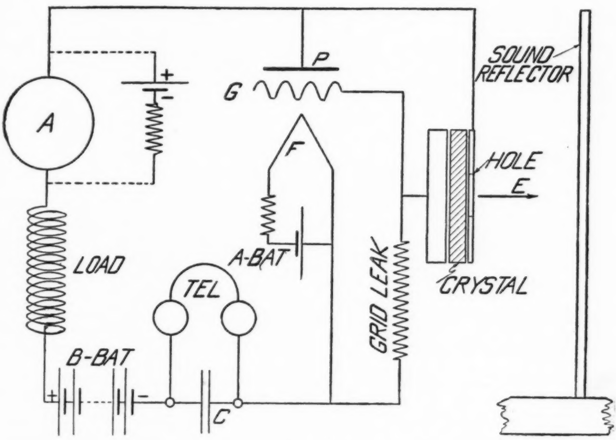

FIGURE 5. The author's type of electric circuit best suited for crystal frequency determined by dimension *E*.

It is also possible to cause the system to oscillate with the period determined by a larger dimension of the vibrator (that is in the direction of the axis *B*).

To attain this vibration of longer period I find that the best circuit is that shown in Figure 6 which is similar to Figure 5 except that the crystal vibrator is connected between the grid and some point below the inductance, as for example, the positive end of the B battery. The telephones, or their equivalent, acting as a choke, and the bypass condenser shunting them, are, in this case, usually necessary to give the proper reaction to make the system oscillate. This arrangement causes the crystal to vibrate with a period determined by the mechanical frequency of the crystal slab, expanding and contracting along the direction of the B axis, so that sound is radiated in this direction.

Other types of piezoelectric oscillating circuits will be described elsewhere.

6. Exploration of Sound Waves.— In Figures 5 and 6 is also shown a sound reflector which may be moved toward or away from the crystal so as to explore the standing sound waves. *No additional apparatus for detecting the sound is necessary, for the reflected sound wave falling on the emitting face of the crystal vibrator, even when the reflector in some cases is at a distance of 300 half waves of sound from the vibrator, reacts on the crystal with sufficient force to cause the current in the milliammeter A to fluctuate visibly in accordance with the phase of arrival of the reflected wave.*

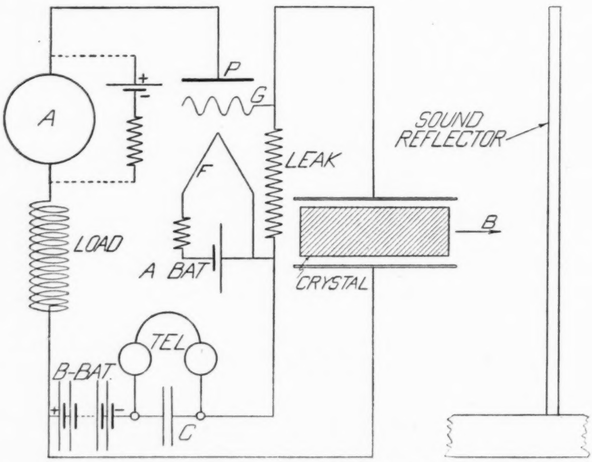

FIGURE 6. Type of electric circuit best suited for crystal frequency determined by dimension B.

When a small dry-battery vacuum tube (UV 199) is used the normal current in A when the crystal is oscillating is about 0.5 milamperes and the fluctuations range from 1.0 milampere (or circuit stops oscillating) when the mirror is close to the crystal down to zero when the mirror is at infinite distance from the crystal. To render the fluctuations more evident for precision measurements, a Weston microammeter, with one division equal to 4 microamperes, is used at A and is shunted by a potentiometer and battery combination to make the normal

current through A small, say 10 divisions (40 microamperes). The fluctuations in the current in A, as the reflector is moved, may then amount to the whole additional scale, 90 divisions, when the reflector is 20 half wavelengths of sound away, and to about 25 divisions when the reflector is 100 half wavelengths away. These magnitudes depend on the frequency and area of radiating face of the crystal. The microammeter gives a sharp maximum at each half wavelength of displacement of the reflector.

It is seen that the apparatus oscillates of itself with a highly constant fixed frequency determined by a mode of mechanical vibration of the crystal plate. It thus produces sound waves and at the same time, by the strength of the plate current, determines the relative phase of the direct and reflected sound waves in air, so that the distance be-

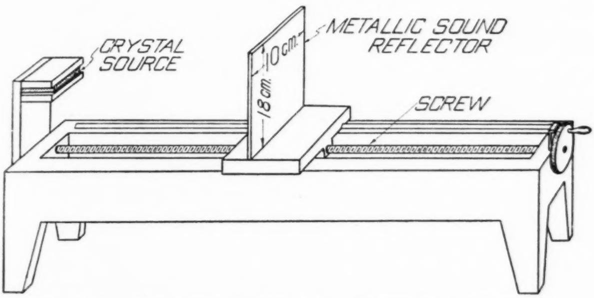

FIGURE 7. Mounting of crystal and mirror with screw drive.

tween successive loops of the standing sound wave for a range of 100 or more half wavelengths may be measured with great precision, by merely changing the distance between the reflector and the radiating face of the crystal, and making proper readings. These distances were changed by a calibrated precision screw, illustrated in Figure 7, by which the mirror could be moved about 50 centimeters and its position read with an accuracy of one one-thousandth of a millimeter, but such accuracy was not ordinarily required or attainable in the location of maxima. A modification of the mounting is shown in Figure 8, in which the mirror and crystal are contained in a gas-tight box.

7. Example of Microammeter Readings Plotted Against Crystal-to-Mirror Distance.— To show the nature of the observations of the standing waves reference is made to Figure 9, which is a

plot of divisions of the microammeter in the plate circuit of the oscillator as ordinates, against the scale reading in millimeters of crystal-to-mirror distance. Three half waves are plotted so that an idea may be had of the accuracy of location of the maxima. In this particular

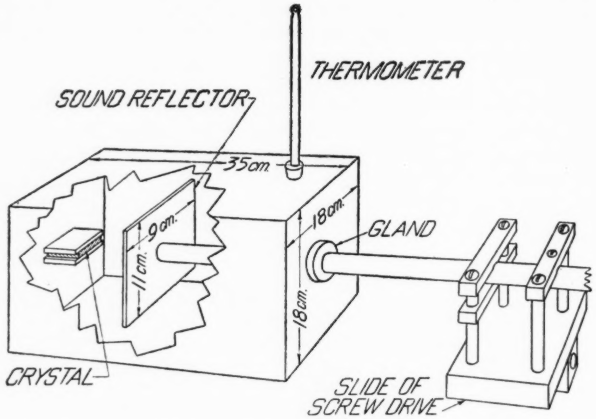

FIGURE 8. Crystal and mirror in brass box for containing gas.

experiment, in which the sound frequency was 98183 cycles per second, the maxima could be located to better than 1/20 millimeter, and since the train of standing waves in this case was explored for 140 millimeters, this degree of precision of locating the maxima gives the

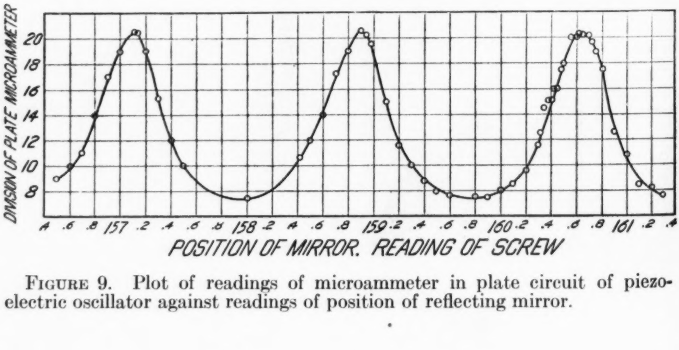

FIGURE 9. Plot of readings of microammeter in plate circuit of piezoelectric oscillator against readings of position of reflecting mirror.

wavelength to about 1/30 of one per cent. In the case of some of the other frequencies a still greater degree of precision was attainable.

For the present purpose it was not necessary to make readings in sufficient number to plot the complete standing wave system. It was only necessary to determine the positions of the mirror for maxima of the microammeter, and to locate accurately such of these positions as were to be used in the calculations. Usually maxima at intervals of five, ten, or twenty half wavelengths were employed.

8. Method of Averaging Observations.— Suppose that we have set the reflecting mirror at a series of positions of maxima schematically represented as *positions* 0, 1, 2,. . . . n, in Figure 10, and let the dis-

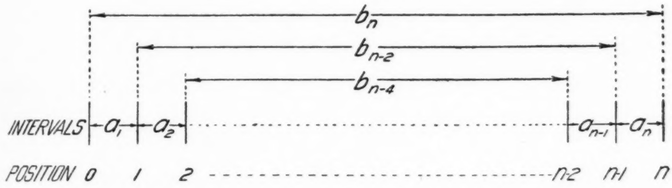

FIGURE 10. Diagram used in discussion of method of averaging.

tances, which we shall call *intervals*, between successive *adjacent positions* be a_1, a_2,. . . . a_n. These intervals are attempts to measure the same quantity x (say), and question arises as to the best method of averaging the intervals to get this quantity x. If we merely take

$$x = \frac{a_1 + a_2 + \,.\,.\, + a_n}{n} \text{ we see that } x = \frac{\text{distance from 0 to } n}{n}\text{; that is}$$

to say, this method of averaging, if employed, makes the result depend on the settings 0 and n alone and ignores all the intervening settings, which might as well not have been made. This method of averaging is evidently not correct.

Assuming that the positions 0, 1,. . . . n are all located with equal accuracy we may obtain the proper method of averaging by regarding the distance between any pair of positions, for example, the position 2 and position 8, as data for determining the fundamental interval, giving to this distance a weight proportional to its length $(8-2=6)$ and averaging for all such pairs of positions, with due precaution to count every pair only once. Starting with position 0 we have dis-

tances 0 to 1 = a_1, 2(0 to 2) = $2(a_1 + a_2)$, etc. Next starting with position 1, we have (1 to 2) = a_2, 2(1 to 3) = $2(a_2 + a_3)$, etc. These are collected in Table I.

TABLE I.

Weighted Distances for Determining Fundamental Interval.

Starting with Position			
0	1	2	...
a_1	a_2	a_3	...
$2(a_1+a_2)$	$2(a_2+a_3)$	$2(a_3+a_4)$	...
$3(a_1+a_2+a_3)$	$3(a_2+a_3+a_4)$	$3(a_3+a_4+a_5)$	...
....	....	....	...
$n[a_1+a_2+\ldots\ldots+a_n]$	$(n-1)[a_2+a_3+\ldots\ldots+a_n]$	$(n-2)[a_3+a_4\ldots\ldots+a_n]$	...
Starting with Position			
....	$n-3$	$n-2$	$n-1$
....	a_{n-2}	a_{n-1}	a_n
....	$2(a_{n-2}+a_{n-1})$	$2(a_{n-1}+a_n)$	
....	$3(a_{n-2}+a_{n-1}+a_n)$		

The sum of all these quantities Σ, say, is

$$\Sigma = [1 + 2 + 3 + \ldots\ldots + n]\ [a_1 + a_2 + \ldots\ldots + a_n]$$
$$+ [2 + 3 + \ldots + n - 1]\ [a_2 + a_3 + \ldots\ldots + a_{n-1}]$$
$$+ [3 + 4 + \ldots + n - 2]\ [a_3 + a_4 + \ldots\ldots + a_{n-2}] + \text{etc.}$$

Now, as in Figure 10, putting

$$b_n = a_1 + a_2 + \ldots\ldots + a_n,$$
$$b_{n-2} = a_2 + a_3 + \ldots\ldots + a_{n-1}, \text{ etc.},$$

and noting that

$$(1 + 2 + 3 + \ldots\ldots + n) = \frac{(1 + n)\,n}{2},$$

$$(2 + 3 + \ldots\ldots + n - 1) = \frac{(1+n)\,(n-2)}{2},$$

we obtain, after dividing out the common factor $\frac{1+n}{2}$,

$$\Sigma = n\, b_n + (n-2)\, b_{n-2} + (n-4)\, b_{n-4} + \ldots\ldots \tag{1}$$

This sum may be otherwise written in the form

$$\Sigma = n\, a_1 + 2\,(n-1)\, a_2 + 3\,(n-2)\, a_3 + \ldots\ldots + n\, a_n \tag{2}$$

If now the fundamental interval is x, which $a_1, a_2, \ldots\ldots. a_n$ are attempts to measure, the value of x may be obtained from (1) or (2) by dividing Σ by the sum of the coefficients of the a's, giving

$$x = \frac{n\, b_n + (n-2)\, b_{n-2} + (n-4)\, b_{n-4} + \ldots}{n + 2(n-1) + 3(n-2) + \ldots + n}, \tag{3}$$

or

$$x = \frac{n\, a_1 + 2\,(n-1)\, a_2 + 3\,(n-2)\, a_3 + \ldots\ldots\ldots\ldots + n\, a_n}{n + 2\,(n-1) + 3\,(n-2) + \ldots\ldots + n}. \tag{4}$$

Equation (3), *or the alternative equation* (4), *gives the weighted mean value of the quantity* x, *of which* $a_1, a_2, \ldots.$ *are the observed values.* Equation (4) is the more convenient and is used below.

9. Temperature Reduction.— For convenience in reducing wavelengths and velocities measured at temperature $t°$ C. to the corresponding values at 0° C., Table II has been compiled.

TABLE II.

Temperature Reduction Table.

$v_0 = v_t \times \theta$, where $\theta = 1/\sqrt{1 + 0.00367t}$.

$t°$ C.	θ Temperature factor	0.1 × diff. Interpolation
15	0.97365	
16	190	−17
17	025	
18	.96857	
19	692	−16.5
20	527	
21	.96365	
22	198	−16.5
23	036	
24	.95875	
25	713	−16.1
26	553	

10. Sample Set of Observations on Wavelength of Sound in Air.— Table III contains a sample set of measurements of the wavelength of sound in air at 205620 cycles per second. With the mirror 3.5 cm. from the crystal a position of maximum deflection of the microammeter was noted. The mirror was then moved twenty half

TABLE III.

Run No. 144. Wavelength of Sound in Air.
Frequency 205620 Cycles per Second.
Steps 20 $\lambda/2$. Humidity 86%.

Temp. degrees C.	Step $20\lambda/2$ cm.	Weight	Weight × step
22.8	1.6812	17	28.580
	1.6805	2 × 16	53.776
	1.6832	3 × 15	75.744
	1.6830	4 × 14	94.248
	1.6767	5 × 13	108.986
22.92	1.6736	6 × 12	120.499
	1.6830	7 × 11	129.591
	1.6830	8 × 10	134.640
	1.6761	9 × 9	135.764
	1.6962	10 × 8	135.696
23.00	1.6672	11 × 7	128.374
	1.6861	12 × 6	121.399
	1.6814	13 × 5	109.298
	1.6730	14 × 4	93.688
22.98	1.6806	15 × 3	75.627
	1.6754	16 × 2	53.613
22.90	1.6829	17	28.609
22.92		969	Σ = 1628.23

$$\lambda = \frac{1628.23}{969 \times 10} = 0.16803 \text{ cm.}$$

Temperature factor = 0.96046

$$\lambda_0 = 0.16138 \text{ cm.}$$

wavelengths farther away. This was found to be a displacement of 1.6812 cm., which is recorded as the first value in the table. The mirror was then displaced a second step of twenty half wavelengths, giving 1.6805, and so on for seventeen steps, encompassing a wave-train of 340 half wavelengths. This table thus epitomizes the exploration of a stationary wave system of 340 maxima. Averaging results as in § 8 we obtain the wavelength as 0.16803 cm. at 22.92° C., which reduced to 0° C. gives $\lambda = 0.16138$ cm.

The data of Table III are for a single run. Five other similar runs were made at this frequency, and the complete set of values of wavelength at 0° C. are recorded as λ_0 in Table IV.

TABLE IV.

Collection of Results of Wavelength and Velocity of Sound in Air at 0° C.

Frequency 205620 cycles per second.

Humidity per cent	Run no.	λ_0 cm.	Residual
87	141	0.16146	0.00015
80	142	.16118	13
79	143A	.16115	14
86	143B	.16137	6
86	144	.16138	7
84	153	.16132	1
Mean 84		0.16131	0.00009

$$\lambda_0 = 0.16131 \pm 0.00003$$
$$v = 331.69 \pm 0.06 \text{ at } 0° \text{ C.}$$

In the third column is the value of λ_0 for each of the six runs, having an average of $\lambda_0 = 0.16131$ cm. In the fourth column are the residuals, which are the amounts by which the individual values differ from their mean. The probable error of the mean E_a computed by the approximate formula

$$E_a = 0.6745 \frac{\text{mean residual}}{\sqrt{n-1}},$$

where n = number of separate values, is 0.00003 cm. Multiplication of λ_0 (reduced to meters) by the frequency gives for the velocity of sound at 0° C. at this frequency, 205620 cycles per second, the value

$$v = 331.69 \pm 0.06 \text{ meters per second.}$$

This result was obtained in free air without any enclosure about the gas. The mean humidity during the several runs was 84 per cent.

In a similar way other high frequencies were employed. For comparison a determination of the velocity of sound in air at the audio frequencies 995.88 and 2987.6 cycles per second was also made. For this purpose a novel method was employed, a description of which follows.

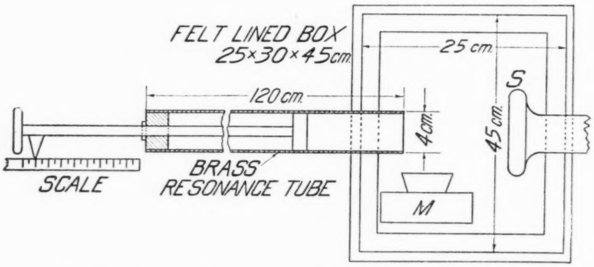

FIGURE 11. Apparatus for measuring velocity of low-frequency sound. S = magnetophone source, M = microphone detector.

11. Low-Frequency Sound Required a Special Method.— The crystal oscillator method with crystals at hand could not be employed at frequencies below about 40000 cycles per second, so that for low frequencies resort was had to a resonance-tube method. This method as ordinarily employed is inaccurate. A modification was introduced that resulted in a considerable improvement. This consisted in the use of a superimposed interference effect that gave a position of practical silence, or null point, in the middle of what would ordinarily be the sound maximum.

12. Null Method.— A sketch of the apparatus for this purpose is given in Figure 11. A brass resonance tube 120 cm. long and 4 cm. internal diameter, and provided with a piston and scale, was employed. The source of sound was a magneto telephone at S, which was driven by a tuning fork of known frequency. The source S, the mouth of the resonance tube, and a microphone receiver M were enclosed in a felt-

lined wooden box, and the distances between them were adjusted so that the sound direct from S to M just neutralized the sound from the tube to M when the sound emitted by the tube was a maximum. This gave a fiducial point of silence in the middle of what would otherwise be a maximum and permitted very accurate settings with sound of frequency 995.88 cycles per second. The microphone M communicated through a transformer with a head telephone receiver in which the observer listened. By means of an electric filter in this circuit, the fundamental frequency of the sound (995.88 cycles per second) could be eliminated and settings could then be made on the harmonic of $3 \times 995.88 = 2987.6$ cycles per second, with which the settings were still more accurate.

13. Sample Sets of Low-Frequency Sound Measurements.—Table V contains a sample set of observations at 995.88 cycles per second analyzed by the method of § 8.

TABLE V.

Run No. 67 in Air at 995.88 Cycles per Second.

Temp. degree C.	Setting at minima in cm.	Mean setting	$\lambda/2$ cm.	Wt.	$\lambda/2 \times$ wt.
21.7	20.32				
	.32				
	.33	20.323	17.274	4	69.096
	37.58				
	.60				
	.61	37.597	17.339	6	104.034
	54.86				
	.96				
	.96				
	.96	54.936	17.307	6	103.842
	72.26				
	.21				
	.26	72.243	17.190	4	68.760
	84.44				
	.47				
	.39	89.433			
				20	345.732

$$\lambda/2 = 345.732 \div 20 = 17.287 \text{ cm.}$$
$$\lambda = 34.573 \text{ cm.}$$
$$\text{Temperature factor} = 0.96250$$
$$\lambda_0 = 33.276 \text{ cm.}$$

Other values of this wavelength λ_0 are collected in Table VI, in which they are given weights proportional to the number of actual settings at each position of the piston.

TABLE VI.

Collection of Results for Frequency 995.88 Cycles per Second.

Humidity per cent	Run no.	λ_0 cm.	Weight	Residuals
36	66A	33.350	1	0.019
36	66B	33.266	1	.065
36	67	33.276	3	.044
35	69	33.320	3	.011
52	72	33.412	3	.081
52	73	33.328	6	.003
55	74	33.340	6	.009
Weighted mean		33.332		0.027

Whence $v = 331.94 \pm 0.07$ m/sec. at 0° C. Not corrected for effect of tube.

TABLE VII.

Collection of Results for Frequency 2987.6 Cycles per Second.

Humidity per cent	Run no.	λ_0 cm.	Weight	Residuals
36	66C	11.105	2	0.007
36	67B	11.110	1	.002
35	68A	11.115	6	.003
35	68B	11.111	3	.001
Weighted mean		11.112		0.003

$v = 331.98 \pm 0.03$ m/sec. at 0° C.

The weighted mean value of velocity of sound in air at 0° C. (Table VI) obtained at this frequency is 331.94 meters per second with a probable error of 0.07 meters per second, uncorrected for effects of the tube.

This apparatus proved to be especially accurate when applied to the measurement of the velocity of the harmonic frequency 2987.6, as is shown by the results in Table VII.

14. Dimensions of Crystal Vibrators.— Figure 12 gives a dimensional sketch of the different crystal vibrators employed. The

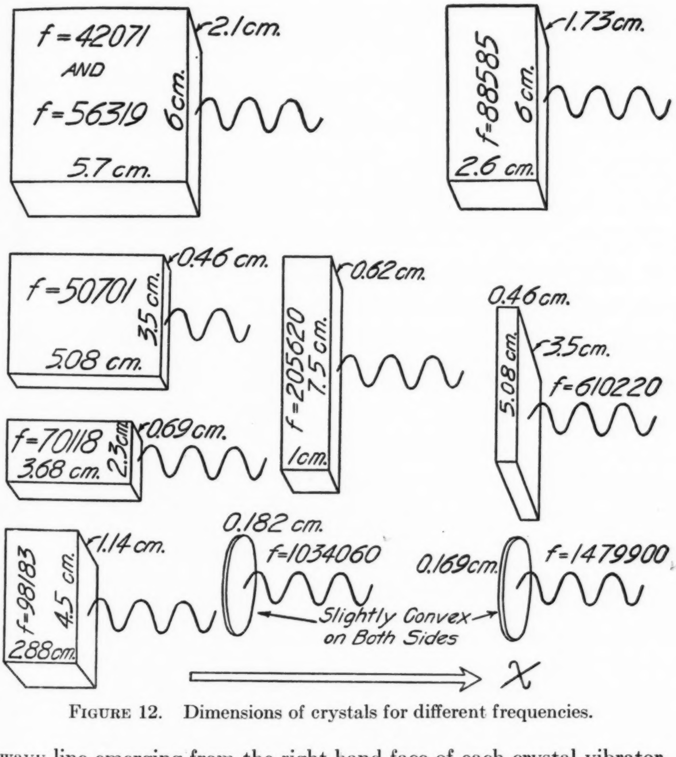

FIGURE 12. Dimensions of crystals for different frequencies.

wavy line emerging from the right-hand face of each crystal vibrator indicates the emitting surface and the direction of radiation of sound. In each specimen the optical axis is vertical in the sketch and the

electric axis is perpendicular to the paper for all except those of frequencies 610220, 1034060, and 1479900, in which the electric axis is in the direction of sound radiation (to the right in the sketch). The frequency, designated by f, is marked on each vibrator. The upper left-hand specimen was used for two different frequencies, which could be differentiated one from the other, and each employed at will, by throwing in or out a suitable condenser in shunt to the inductance marked "*load*" of Figure 6. This same vibrator was originally somewhat longer in the dimension running right and left in the sketch and then gave the frequency 41009 cycles per second. After being used at this frequency it was ground down to give the frequencies indicated in the sketch.

Table VIII contains the frequencies of the several crystal vibrators and also the dimension x (in direction right to left in sketch). This x

TABLE VIII.

Relation of Frequency to Dimension x.

Axis of radiation	x Determining dimension in cm.	f cycles per sec.	fx
B	5.7	42071	23.98×10^4
		56319	32.10
B	5.08	50701	25.76
B	3.68	70118	25.80
B	2.61	88585	23.12
B	2.88	98183	28.29
B	1.00	205620	20.56
B	0.46	610220	28.07
E	0.182	1034060	18.82
E	0.169	1479900	25.01
		Average	25.14×10^4

is the dimension in the direction of propagation of the sound. The product of f in cycles per second and x in centimeters is contained in the last column. The average of this product is about 25×10^4, and this may be employed as a very rough guide in cutting the crystal vibrators for a required frequency.

After the crystals are cut the vibrators are ground with coarse carborundum powder and finished with fine carborundum, fine emery, and in the case of thin specimens with rouge, to such fineness of surface as may be required for efficient oscillation. Repeated measurements of frequency during grinding enable a skillful operator to attain a desired frequency to better than 1/100 of 1 per cent, if standards of that precision are available.

15. Standards of Frequencies.— This subject is treated in the previous paper [5] to which reference is made for details. A standard tuning fork, here designated fork "*S*," made by the Westein Electric Company, was used to operate electrically a high-speed siphon recorder made by the General Radio Company, and was thus chronographed four times for a period of 300 seconds each and once for a period of 250 seconds, and was found to have a frequency of 49.916 at 18.2° C., with a probable error of 1/400 of 1 per cent. Immediately preceding and following this operation, the 20th harmonic of the fork was observed to beat 2.44 times per second with a second fork, driven by a vacuum tube circuit, as shown in the previous paper. This second fork, here designated fork "*P*," was thus found to have a frequency

$$f_{\text{fork } P} = 20 \times 49.916 - 2.44 = 995.88 \pm 0.02$$

at 18° C.

Next a standard piezoelectric crystal oscillator permanently mounted was calibrated in terms of fork *P*, by the use of an electric oscillator and resonant-circuit wavemeter as intermediary. The wavemeter was calibrated by setting it to resonance with the electric oscillator when the latter was at a beat zero with the 6th to the 18th harmonic of fork *P*, and immediately thereafter the readings of the wavemeter were again taken at resonance with the electric oscillator when each of the harmonics of the electric oscillator between the 25th and the 58th was at beat zero with the standard piezoelectric crystal oscillator. By careful interpolation this gave

$$f_{713} = 420710 \pm \frac{1}{200} \text{ per cent.}$$

[5] Pierce, Proc. Am. Acad. of Arts and Sciences, **59,** No. 4 (1923).

This crystal oscillator will be known as No. 713, which is approximately its electric wavelength.

In terms of this crystal and its harmonics and multiples, the entire wavemeter was now carefully calibrated for the range of frequencies between 6000 and 6,000,000.

16. Measurement of Frequencies of Crystal Vibrators.—Immediately following or preceding the exploration of the sound wave system the frequency of each crystal vibrator was measured by one or more of the following methods:

I. *Harmonic Bracketing with Standard Crystal.* This consists of obtaining a wavemeter setting on an electric oscillator at beat zero with some harmonic of the unknown crystal X and then obtaining, on both sides of this wavemeter setting and as near to it as possible, a wavemeter setting on the electric oscillator at beat zero with some two harmonics of the standard crystal No. 713. Interpolation gives the wavemeter correction and the value of the frequency of the unknown.

II. *Audio-Frequency Beats.* Intercomparison of crystal vibrators was made in some cases by measuring on an audio-frequency meter the beat frequency of the fundamental of one crystal with some harmonic of another crystal.

III. *Superheterodyne Method.* When the beat frequency of II was above audibility, a third oscillator frequency could be made to beat with the rectified beat frequency of the two crystals. Measurement of the third frequency (which was a difference) could then be made with sufficient accuracy by the wavemeter.

IV. *Subheterodyne.* When the beat frequency of II was too low for audibility it could be made audible by an oscillator beating with both crystals (say 1000 per second with one and 1002 per second with the other), then the beating of the two audio frequencies with each other could be counted (as 2 beats per second).

V. *Direct Check of Crystals against Tuning Fork P.* This is a method similar to that used in § 13.

These various methods will be designated in the tables of results. As an example of Method I an electric oscillator beating with the fundamental of a crystal X and two harmonics of the oscillator beating with crystal No. 713 gave readings as in Table IX. The fundamental wavelength of No. 713 is 713.08 meters, based on 3×10^8 meters per second as the velocity of the waves, so that by regarding λ of column three as approximately correct it is found that n of the fourth column is 5 and 14/3, as entered. These last two

numbers are then multiplied by 713.08 and entered as $\lambda_{correct}$ for No. 713. A comparison of columns three and five gives the correction increments 0.3 and 0.9 entered as δ for No. 713. Interpolation between these gives $\delta = +0.8$ for X, which added to the observed λ gives $\lambda_{correct} = 3386.6$, for which the frequency is 88585. Note that the wavemeter calibration was in error less than 1/30 of 1 per cent in the range of Table VIII.

TABLE IX.

Determination of Frequency of Crystal X.

Crystal	Condenser divisions of W.M. No. 14 Coil E	λ Meters by calibration table	n	λ correct	δ	f
X	10.409 10.410 10.410	3385.8		3386.6	0.8	88585
713	11.530 11.532 11.530	3565.1	5	3365.4	.3	
713	10.048 10.047 10.047	3326.8	14/3	3327.7	.9	

17. Simultaneous Exploration and Frequency.— In some cases two crystal vibrators were mounted one above the other on the sound-exploring apparatus, their two sound patterns explored simultaneously, and the beat frequency between one of the vibrators and some harmonic of the other continuously observed by an audio-frequency meter. This was a safeguard against possible changes in frequency or in the effects of temperature or composition of the air.

18. Negligible Change of Frequency of the Crystal Oscillator Due to Reaction of Reflected Sound.— The reflected sound changed the plate current of the piezoelectric oscillator. The crystal of frequency 42071 was made to have this value so that its 10th harmonic would have the frequency of the standard crystal No. 713, which is 420710. It was found that by slight adjustment of the clearance of the electrodes of 42071 its 10th harmonic gave 1.4 beats

per second with No. 713, when the sound reflector was adjusted at a position such that the plate ammeter of the oscillator was at a minimum. When now the reflector was moved along the sound pattern in space until the ammeter deflection was a maximum the beat frequency was 8 per second. It thus appears that the effect of the mirror was a change of frequency of 6.6 cycles per second in 420710, or 0.66 cycles per second in 42071, which is 1/700 of 1 per cent.

19. Negligible Effect of Humidity on the Velocity of Sound.—According to standard tables [6] the density of air at 20° C. is about 0.4 per cent less at 50 per cent humidity than for dry air at the same temperature. If the moisture affected the velocity of sound only by changing its density we should expect that the velocity in air at 50 per cent humidity would be about 0.2 per cent more than the velocity in dry air. On the contrary, I have not been able to detect any effect of humidity on the velocity of sound, although the following careful measurements were made. In the metal box shown in Figure 8 seven runs were made with air dried by phosphorus pentoxide so that a hygrometer read zero humidity and seven similar runs were made with the air at 50 per cent humidity. This was done at a frequency of 98183 cycles per second and gave for the wavelength reduced to 0° C. the values in Table X. The effect of humidity up to 50 per cent is seen to be less than the probable error which is less than 1/100 of 1 per cent.

It should be noted, however, that in this experiment in order to obtain dry air in the metallic sound chamber, it was found necessary to remove the felt lining, so that we have not a sound pattern determined solely by the distance from the crystal to the mirror. The velocity of sound at 0° C., calculated from the above wavelength values and the frequency, comes out to be 331.56, whereas in open air or in a box lined with nonreflecting material the velocity at this frequency is found to be 331.77. The absence of lining of the box thus makes in the absolute value of the velocity an error of 1/15 of 1 per cent, but it is not believed that this could have any effect in making the apparent velocity the same in dry air and in damp air.

Many other experiments made in the course of this research have failed to disclose any effect of humidity and indicate that the effect of humidity up to 80 per cent on the high frequency velocity of sound is below 1/50 of 1 per cent. This is a novel result contrary to existing ideas.

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[6] Kaye and Laby.

TABLE X.

Sound Wavelength at 0° C. at 98183 Cycles per Second for Dry Air and Moist Air.

Dry air	50% Humidity
0.33759 cm.	0.33780 cm.
.33774	.33760
.33777	.33776
.33770	.33789
.33766	.33762
.33775	.33753
.33770	.33765
Average	
0.33770 cm.	0.33769 cm.
Probable error	
0.00001 cm.	0.00003 cm.

20. Results on Velocity of Sound in Air at 0° C.— Table XI gives the results on air at 0° C. All of these values except those at $f = 995.88$ and $f = 2987.6$ were obtained in free air without any container whatever.

TABLE XI.

Velocity of Sound in Air at 0° C.

f Frequency cycles per second	v Velocity meters per second at 0° C.	Probable error in meters due to errors in wavelength	Probable error in meters due to errors in frequency	Method of determining frequency
995.88*	331.94*	0.07	0.02	Fork *P*
2987.6*	331.98*	.03	.02	Fork *P*
41009	332.45	.03	.07	Beats with 205620
42071	332.37	.06	.01	Beats with No. 713
50701	332.47	.06	.07	Beats with 205620
56319	332.32	.08	.07	Bracketed by No. 713
70118	331.98	.02	.02	Beats with No. 713
88585	331.97	.03	.05	Bracketed by No. 713
98183	331.77	.07	.02	Checked *vs.* Fork *P*
205620	331.67	.06	.07	Bracketed by No. 713
610220	331.81	.07	.06	Bracketed by No. 713
1034060	331.76	.03	.05	Superheterodyne
1479900	331.64	.05	.04	Bracketed by No. 713

* Measured in a resonance tube and uncorrected for the diameter of the tube.

These results are plotted in Figure 13.

21. Effect of Frequency on the Velocity of Sound in Air.— For the present omitting from consideration the two values at $f = 996$ and $f = 2988$ which were measured in a tube, and for which there may be an uncertain tube correction, we see that between $f = 50000$ and $f = 100000$ the velocity falls from $v = 332.47$ m/sec. to $v = 331.78$ m/sec., which is a decrease of 1/5 of 1 per cent. The velocity falls further to $v = 331.67$ at $f = 200000$, then seems to rise again to $v = 331.81$ m/sec. at $f = 600000$, and thence to fall to $v = 331.64$ at

$f = 1500000$. It is to be noted, however, that in the range from $f = 100000$ to $f = 1500000$ the straight broken line departs from the observed values by a maximum of about 1/40 of 1 per cent or 0.09 m/sec., which is about the probable error of observations. The actual existence of the maximum at $f = 600000$ may, therefore, be questioned.

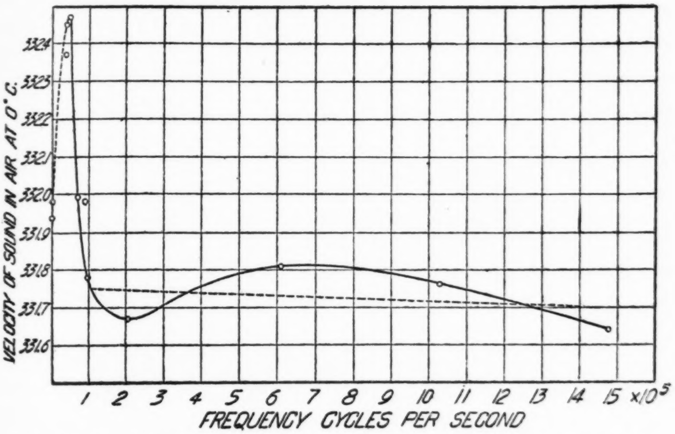

FIGURE 13. Velocity of sound in air at 0° C. Plotted against frequency.

22. Precautions in Measurements.— To make sure that the frequencies and experimental conditions did not vary between observations, one set of measurements of velocity at $f = 50701$ and $f = 205620$ was made with the two crystals mounted one above the other so that their sound patterns could be explored simultaneously. Each of the crystals was provided with its own vacuum tube, load inductance, and microammeter. The two load inductances were placed in inductive relation to each other so that in a telephone in circuit with the crystal $f = 50701$ beats could be heard between the harmonic $f = 4 \times 50701$ of one crystal and the fundamental $f = 205620$ of the other. These beats, measured on a special bridge-type of audio-frequency meter, remained between 2835 and 2837 cycles per second so that, if the indicated frequencies are correct, we should have

$$4 \times 50701 + 2836 = 205620$$

$$\text{i.e.} \qquad 205640 = 205620$$

which is in agreement to within 1/100 of 1 per cent. While this agreement of frequency was being maintained, the sound wavelength measurements gave $\lambda_0 = 0.65555$ cm. at $f = 50701$, and $\lambda_0 = 0.16129$ cm. at $f = 205617$, whence the calculated velocities of sound at 0° C. are 332.37 m/sec. at $f = 50701$ and 331.64 m/sec. at $f = 205617$. These values show that the higher frequency gives the lower velocity by an amount considerably greater than the errors of observations. The results at this frequency in Table XI contain this run averaged with certain others not made simultaneously.

Another precaution consisted in the interspersing of readings at one frequency with those of another so that gradual changes or improvements of methods should not introduce differences in the results.

23. Effort to Extend the Observations to Higher Frequencies.—With a crystal vibrator giving 3,000,000 oscillations per second, effort was made to obtain the velocity of sound in air, but no reactive effect could be obtained by moving the reflector. Either the crystal was insensitive to reaction, or, what is more probable in view of the results for CO_2 (see § 25), the air was opaque to these frequencies of three million cycles per second.

24. Concerning Previous Measurements of Sound Velocity at High Frequencies.—The only previous measurements of the velocity of sound at high frequencies were by E. Dieckmann [7] in 1908 by the use of a Poulsen Arc as source, a transmission grating for dispersion, and a suspended vane for detector. Frequencies were measured by a wavemeter. The medium was illuminating gas, in which Dieckmann obtained a velocity of 456. m/sec. independent of frequency between $f = 78000$ and $f = 780000$ cycles per second, but the method employed was not capable of high precision.

25. Measurement of the Velocity of Sound in CO_2 at High Frequencies.—The metallic container illustrated in Figure 8 was lined with cotton wool and after long flushing with CO_2 gas from a commercial cylinder of liquid CO_2, the velocity of sound was measured for three frequencies with the results given in Table XII. The water vapor present, which is the chief impurity in commercial CO_2, represented a humidity of 50 per cent at 20° C.

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[7] E. Dieckmann, Ann. der Phys., **27**, 1066 (1908).

TABLE XII.

Velocity of Sound in CO_2 (Commercial) at 0° C.

Frequency cycles per second	Velocity meters per second at 0° C.	Probable error in meters per second
42071	258.82	0.08
98183	258.94	.16
205620*	260.15	.23
1034060	opaque	

* Highly absorbed.

These results are plotted in Figure 14, and show an increase of velocity with increasing frequency from v = 258.8 at f = 42000 to 260.2 at f = 206000. This work with CO_2 is incomplete and further measurements are in progress. In addition to the relatively large change of velocity with frequency this experiment with CO_2 revealed also a very large absorption at the higher frequencies, as is discussed in the next section.

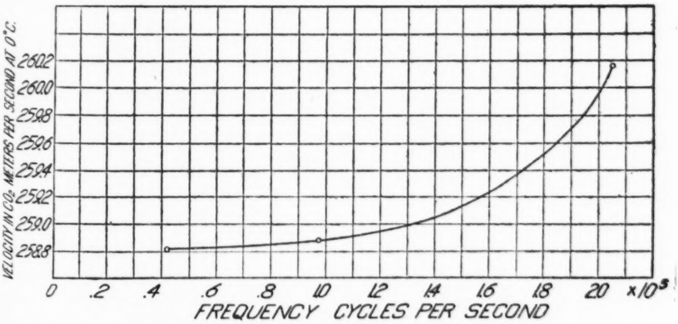

FIGURE 14. Velocity of sound in CO_2 at 0° C. Plotted against frequency.

26. Absorption of High-Frequency Sound by CO_2.— At f = 1034060, CO_2 was found to be practically opaque. No reaction

by the reflected wave could be observed even when the mirror was almost in contact with the crystal. At $f = 205620$ the absorption was very great, as the following data show. Figure 15 is a plot of divisions deflection of the microammeter of the plate circuit of crystal $f = 205620$ against crystal-to-mirror distance in CO_2, at 22.7° C. The distance is in millimeters uncorrected for a slight error of the screw. It is seen that the deflection in going from the maximum at 4 mm. distance to the minimum at 4.4 mm. distance drops from 9.2 divisions to 3.5 divisions, representing an amplitude of reaction by the reflected sound equivalent to 5.7 microammeter divisions.

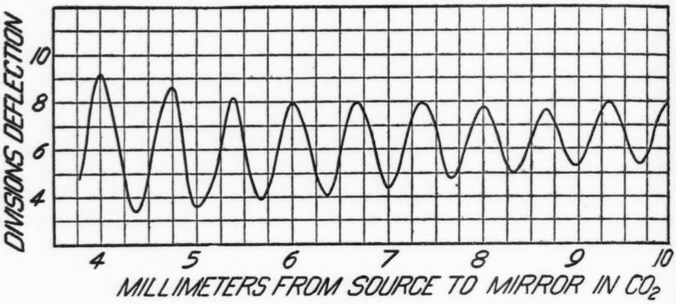

FIGURE 15. Stationary wave system in CO_2.

Immediately after these readings were taken, the CO_2 was removed from the sound box (of Fig. 8) and air substituted. It was then found that with air in the box the reaction due to the reflected wave gave an amplitude of deflection of 10 milliammeter divisions when the crystal-to-mirror distance was 204 mm., which was the greatest distance that the dimensions of the apparatus would permit. We thus have the result that the double transmission through 4 mm. of CO_2 diminishes the sound intensity at $f = 205620$ more than the double transmission through 204 mm. of air. Let us note that the width up and down of the radiating crystal was 6.2 mm. (see Fig. 12) so that in the case of the CO_2, where the mirror was only 4 mm. from the crystal, the departure from a plane wave condition was very small and all the diminution of amplitude was due to nontransmission by the gas; while in the case of the air the mirror was so far from the crystal (204 mm.) that the effect of the departure from a plane wave would be marked, and the diminution of amplitude would be due to

absorption and to divergence as well. Thus the absorption of sound at this frequency $f = 205620$ in CO_2 is more than 80 times and may be several hundred times as great as that of air.

At $f = 98183$ a similar experiment shows that 17 mm. of CO_2 reduces the reflected intensity as much as 68 mm. of air, making the absorption of CO_2 at least 4 times that of air. Further experimental work needs to be done on this subject.

27. Concerning Effect of Box Enclosing Gas in Modifying Standing Wave System.— It was pointed out in § 19 that the sound box of Figure 8 when not lined with absorbing material, although the box is enormous compared with the wavelength of the sound, reduced the apparent velocity of sound by 0.6 per cent at $f = 98183$. On the other hand with the box lined with cotton wool 3 cm. thick, the velocity at $f = 205620$ measured in air in the box was found to be 331.70 m/sec., which within the limit of error checks the value 331.77 of Table XI in free air at this frequency. In making absolute measurements the lining of the box is important and should be studied. Cotton was used rather than hair felt, as it was thought to be less hygroscopic, but a careful comparison of different materials has not been made. Incidentally glass wool in considerable thickness was found to be transparent to high-frequency sound.

28. Discussion of Velocity of Sound in Air at Low Frequency. — The direct design of the present research to measure the stationary wave system of sound in free air could not be employed, with the crystal oscillators at hand, at frequencies below $f = 40000$. Resort was had to a resonance tube method, but, although consistent and accurate readings were obtained, the uncertainty of the tube correction to be applied makes it advisable to refer briefly to results obtained by others.

Work of T. C. Hebb. By an ingenious attempt at direct measurement of the wave system in the free air of a large room, T. C. Hebb[8] in 1905 obtained $v = 331.29$ at $f = 2377$.

Explosion and String Galvanometer Registration. By this method E. Esclangon[9] in France in 1917–18, and v. Angerer and Ladenburg[10] in Germany in 1916–18 at sufficiently great distances from the source obtained respectively $v = 330.9$ and $v = 330.78$ as the velocity of

8 Phys. Rev., **20,** 89 (1905).
9 Comptes rendus, **168,** 165 (1919).
10 Ann. der Physik, **66,** 294 (1921).

sound in dry air at 0° C. I find in a paper by McAdie[11] a reference to a result by D. C. Miller of the value of $v = 330.8$, but I have not found any publication by Professor Miller of this value. If this were also obtained by the same method of explosion and galvanometer registration, we have good evidence for the correctness of 330.8 m/sec. as the speed of explosive waves of low intensity.

Resonance Tube Method. The well-known methods using the resonance tubes give a measured wavelength dependent on the diameter of the tube. Helmholtz taking account of viscosity, and Kirchhoff introducing also the effect of the communication of heat to the walls of the tube, both derived the equation

$$v' = v\left(1 - \frac{c}{d\sqrt{\pi f}}\right), \tag{5}$$

in which

f = frequency,
v = velocity in free air,
v' = " " a pipe of diameter d,
c = a constant.

Very discordant results exist as to the applicability of this formula and as to the value of the alleged constant c. If we assume that my two measurements in the tube at $f = 995.88$ and $f = 2987.6$ are both exact, and that equation (5) is correct, and that the velocity is the same at both frequencies, we may eliminate and obtain

$$v = 332.04 \quad \text{m/sec. at } f = 996 \text{ and } f = 2988.$$

This result is larger than that obtained by other experimenters by a somewhat similar use of equation (5), for example: Stevens[12] who in 1902 obtained $v = 331.32$ and recently Grüneisen and Merkel[13] at the Reichsanstalt who obtained $v = 331.57$, by, however, neglecting all of the observations at frequencies below $f = 3480$, which would have given, if kept, a much larger average velocity. In fact their values between $f = 1390$ and $f = 2780$ would have given $v = 331.98$ if these frequencies had been kept and treated as their remaining data were.

11 Annals of the Observatory of Harvard College, **86,** 107 (1923).
12 Ann. der Phys., **7,** 285 (1902).
13 Ibid., **66,** 293 (1921).

29. Conclusions Regarding Low-Frequency Velocity.— We may hence conclude that the velocity of *explosive waves* of small intensity as registered by galvanometer deflections is 330.8 m/sec. and that the low-frequency velocity measured by standing waves is probably between 331.29 and 332.1 meters per second. The uncertainty surrounding this last value leaves the beginnings of the curve of Figure 13 in doubt, but it seems probable that the dotted line between $f = 0$ and $f = 40000$ represents the real course of the curve.

30. Summary.— I. A simple method is described for (*a*) producing high frequency sound of constant pitch, (*b*) detecting the interference between direct and reflected waves by the reactive response at the source, (*c*) exploring the sound pattern, and (*d*) interchecking the frequencies employed.

II. A null method of measuring the audio-frequency velocity of sound in resonance tubes is described.

III. The velocity of sound in free air and CO_2 has been determined for a range of frequencies between 40000 and 1500000 cycles per second for air and between 400000 and 205000 cycles per second for CO_2. A hitherto undetected change of velocity with frequency has been discovered, as shown in Figure 13 and Figure 14.

IV. Large absorption in CO_2 for frequencies above 205000 cycles per second is shown.

V. The effect of humidity of the atmosphere on velocity of sound is found to be negligible at high frequencies.

31. Theoretical Significance.— It is hoped that additional data may be collected and that at some later date the entire theoretical significance of the results may appear. In the meanwhile the initial increase of the velocity of sound with increase of frequency may be looked upon as due to heat conduction by the gas. In other words, the expansion and compression is not adiabatic for frequencies below 50000 cycles per second for air and below 205000 cycles per second for CO_2. With increase of frequency up to these limits the motion becomes more nearly adiabatic with a resulting increase of velocity.

In the case of CO_2 absorption seems to stop the observations while the curve of velocity against frequency is still rising, but to make sure of this values at frequencies between $f = 100000$ and $f = 200000$ should be obtained. The high absorption in CO_2, if carefully studied, should be of great significance.

In the case of air the frequency of maximum velocity is about

50000 cycles per second and at frequencies higher than this the reduction of velocity may be due to an approach of the period of the sound to the time between collisions of the molecules or atoms of the gas. The existence of two maxima for air (if that at $f = 600000$ is real) may in some way be associated with the two principal ingredients of the air.

Cruft Laboratory, Harvard University,
Cambridge, Massachusetts.
June, 1924.